

The directed flow maximum near $c_s = 0$

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We investigate the excitation function of quark-gluon plasma formation and of directed in-plane flow of nucleons in the energy range of the BNL-AGS and for the $E_{Lab}^{kin} = 40A$ GeV Pb+Pb collisions performed recently at the CERN-SPS. We employ the three-fluid model with dynamical unification of kinetically equilibrated fluid elements. Within our model with first-order phase transition at high density, droplets of QGP coexisting with hadronic matter are produced already at BNL-AGS energies, $E_{Lab}^{kin} \simeq 10A$ GeV. A substantial decrease of the isentropic velocity of sound, however, requires higher energies, $E_{Lab}^{kin} \simeq 40A$ GeV. We show the effect on the flow of nucleons in the reaction plane. According to our model calculations, kinematic requirements and EoS effects work hand-in-hand at $E_{Lab}^{kin} = 40A$ GeV to allow the observation of the dropping velocity of sound via an *increase* of the directed flow around midrapidity as compared to top BNL-AGS energy.

The theory of strong interactions, QCD, exhibits a thermodynamical phase transition to a so-called quark-gluon plasma (QGP) at high energy density [1]. To achieve densities far beyond that of the nuclear ground state one collides heavy ions at high energies [2–7]. If the equation of state (EoS) exhibits no anomalous structure, one expects that the volume and life-time of the QGP increase with energy.

Due to non-equilibrium effects, however, higher impact energies do not necessarily yield a larger amount of energy deposition in the central region. One-fluid dynamical calculations of the compression neglect initial non-equilibrium processes and predict sharp signals for the phase transition to the QGP [2–7]. How strongly these predictions are washed out by non-equilibrium effects has to be investigated via excitation functions. Therefore, the energy range from BNL-AGS energies ($2 - 11A$ GeV) to CERN-SPS energies (previously only $160 - 200A$ GeV) is highly interesting to study the EoS of nuclear matter, and possibly the onset of QGP formation. In particular, data at $40A$ GeV has been taken recently at the CERN-SPS to complete the already existing data [8–10] on the excitation function of collective flow.

Motivated by the fact that the proton rapidity distribution in high energy pp reactions is strongly forward-backward peaked [11], we developed a fluid-dynamical model in which the projectile and target nucleons constitute *distinct* fluids. The continuity equations for the energy-momentum tensor and net baryon current, $\partial_\mu T_i^{\mu\nu} = S_i^\nu$, $\partial_\mu N_i^\mu = S_i$, are solved for each of the fluids i . The couplings S_i^ν , S_i between those fluids are obtained from a parametrization of binary NN collisions [12], which leads to a gradual deceleration instead of instantaneous stopping as in the one-fluid model. In the three-fluid model [13,14] we also consider the newly produced particles around midrapidity as a distinct fluid, which we call “fireball”, as those particles populate a distinct rapidity region as well (at early times!). The assumption of local equilibrium is imposed within each fluid, but the total energy-momentum tensor $T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} + T_3^{\mu\nu}$ does *not* need to be that of an ideal fluid.

However, after several collisions projectile and target nucleons are stopped, and local thermal equilibrium establishes between projectile and fireball fluid or between target and fireball fluid, respectively, and finally for all three fluids. The respective fluids i and j are then unified by adding their energy-momentum tensors and net-baryon currents at the corresponding space-time point, $T_i^{\mu\nu}(x) + T_j^{\mu\nu}(x) = T_{\text{unified}}^{\mu\nu}(x)$, $N_i^\mu(x) + N_j^\mu(x) = N_{\text{unified}}^\mu(x)$. Common values for e , p , ρ and $u^\mu \equiv \gamma(1, \mathbf{v})$ are then obtained from $T_{\text{unified}}^{\mu\nu} = (e + p) u^\mu u^\nu - p g^{\mu\nu}$ and $N_{\text{unified}}^\mu = \rho u^\mu$, and the given EoS $p = p(e, \rho)$. The dynamical unification allows friction-free ideal expansion of equilibrated fluids.

The initial condition corresponds to two approaching but still separated nuclei in the ground state. The model calculation treats the compression (deceleration) stage as well as the subsequent expansion. The fluid-dynamical equations are solved in 3+1 dimensions, cf. [13] for details. The EoS employed here exhibits a first order phase transition to a QGP [6]. The hadronic phase consists of nucleons interacting via relativistic mean-fields [15], plus thermal

pions. The QGP phase is described within the framework of the MIT-Bag model [16] as an ideal gas of u and d quarks and gluons, with a bag parameter $B^{1/4} = 235$ MeV, resulting in a critical temperature $T_c \simeq 170$ MeV at $\rho = 0$, while the critical baryon-chemical potential is $\mu_c = 1.8$ GeV at $T = 0$. The first order phase transition is constructed via Gibbs' conditions of phase coexistence.

Fig. 1 shows the average isentropic velocity of sound, where

$$c_s^2 = \frac{\partial p}{\partial e} \Big|_{s/\rho}, \quad (1)$$

and the fraction of baryon charge in each phase at various energies. Throughout the manuscript, averages on fixed CM-time hypersurfaces employ the time-like component of the net-baryon four-current as the weight-function.

The formation of plasma droplets starts already at BNL-AGS energies. However, the baryon density ρ in this energy domain is large and $\langle c_s \rangle$ is not very small. Fig. 2 shows the pressure as function of energy density at fixed specific entropy. The values $s/\rho = 10, 20$ correspond to $E_{Lab}^{kin} = 10A$ GeV and $40A$ GeV [17]. The derivative ($= c_s^2$) is only small within the mixed phase if the specific entropy is large (hot matter). Thus, the response of the highly excited matter to isentropic density gradients is *not* weak at BNL-AGS energies. However, at the higher CERN-SPS energies, $E_{Lab}^{kin} \geq 40A$ GeV, an extended time-interval where $\langle c_s \rangle$ is small *does* exist.

From the above it is also clear that the previously predicted [6,7,18] slow rehadronization of mixed phase can only occur at rather high specific entropy, and thus bombarding energy, but probably not at BNL-AGS energies. Indeed, Fig. 1 shows that the maximum fraction of mixed phase occurs always around a CM-time of $t \simeq 7$ fm, despite the different energy scales. Note also that non-equilibrium effects in the early stage of the reaction limit the gain of QGP; it does not increase dramatically from $E_{Lab}^{kin} = 40A$ GeV to $160A$ GeV.

The EoS $p = p(e, \rho)$ is usually investigated by studying various flow patterns [5–7,19–24]. We shall discuss here how the directed in-plane flow, formerly called the *bounce-off* [3,9], “responds” to the behavior of $\langle c_s \rangle$.

Fig. 3 shows the flow pattern obtained at $E_{Lab}^{kin} \simeq 8A$ GeV, at a time where we expect departure from ideal flow in the central region. We observe almost no net p_x around midrapidity ($|p_{long}| < 1$ GeV, say). This is due to the expansion into the direction orthogonal to the normal directed flow, resulting from the geometrical shape of the hot and dense region in coordinate space [23,24]. Obviously, the baryon number can only evolve towards larger isotropy in momentum space because $\langle c_s \rangle$ is *not* small at this energy.

At $E_{Lab}^{kin} = 40A$ GeV, $\langle c_s \rangle$ is less than 0.2 when the phase coexistence region is reached from above. This *prevents* more isotropic redistribution of the baryon number in momentum-space, cf. Fig. 4. The distribution is clearly very different from that shown in Fig. 3, with almost no baryons in the upper left or bottom right quadrants where $p_x \cdot p_{long} < 0$.

At even higher energies, $E_{Lab}^{kin} = 160A$ GeV and more, the average p_x per baryon becomes small again (around midrapidity) for purely kinematical reasons [6,21,24], as can also be observed in the data [10]. That behavior is rather insensitive to the EoS.

In summary, we investigated the excitation function of QGP formation in relativistic heavy-ion collisions. Within our model we find that droplets of QGP coexisting with hadronic matter are already produced at the top BNL-AGS energy, $E_{Lab}^{kin} \simeq 10A$ GeV. However, the average speed of sound in the dense baryonic matter does not drop very much, even in case of a first-order phase transition. As a consequence, expansion of matter is not inhibited and we find an almost isotropic momentum distribution in the reaction plane. This is consistent with experimental results obtained for Au+Au collisions at $E_{Lab}^{kin} = 2 - 10A$ GeV [8], showing that the net in-plane momentum diminishes as beam energy increases towards top BNL-AGS energy.

Furthermore, we showed that the forthcoming results of the Pb+Pb reactions at $E_{Lab}^{kin} = 40A$ GeV will test whether the picture of hot QCD-matter as a heat-bath with small isentropic speed of sound (mixed phase) is indeed applicable to heavy-ion collisions in this energy domain. *If* it holds true, our model calculation predicts an increase of the directed net in-plane flow around midrapidity as compared to top BNL-AGS energy, and a clearly non-isotropic momentum distribution around midrapidity. Due to the phase transition, pressure gradients along isentropes are too small to work towards a more isotropic momentum distribution. Thus, those reactions can shed light on the (non-?)existence of a local maximum of directed flow, related to a substantial decrease of the isentropic speed of sound.

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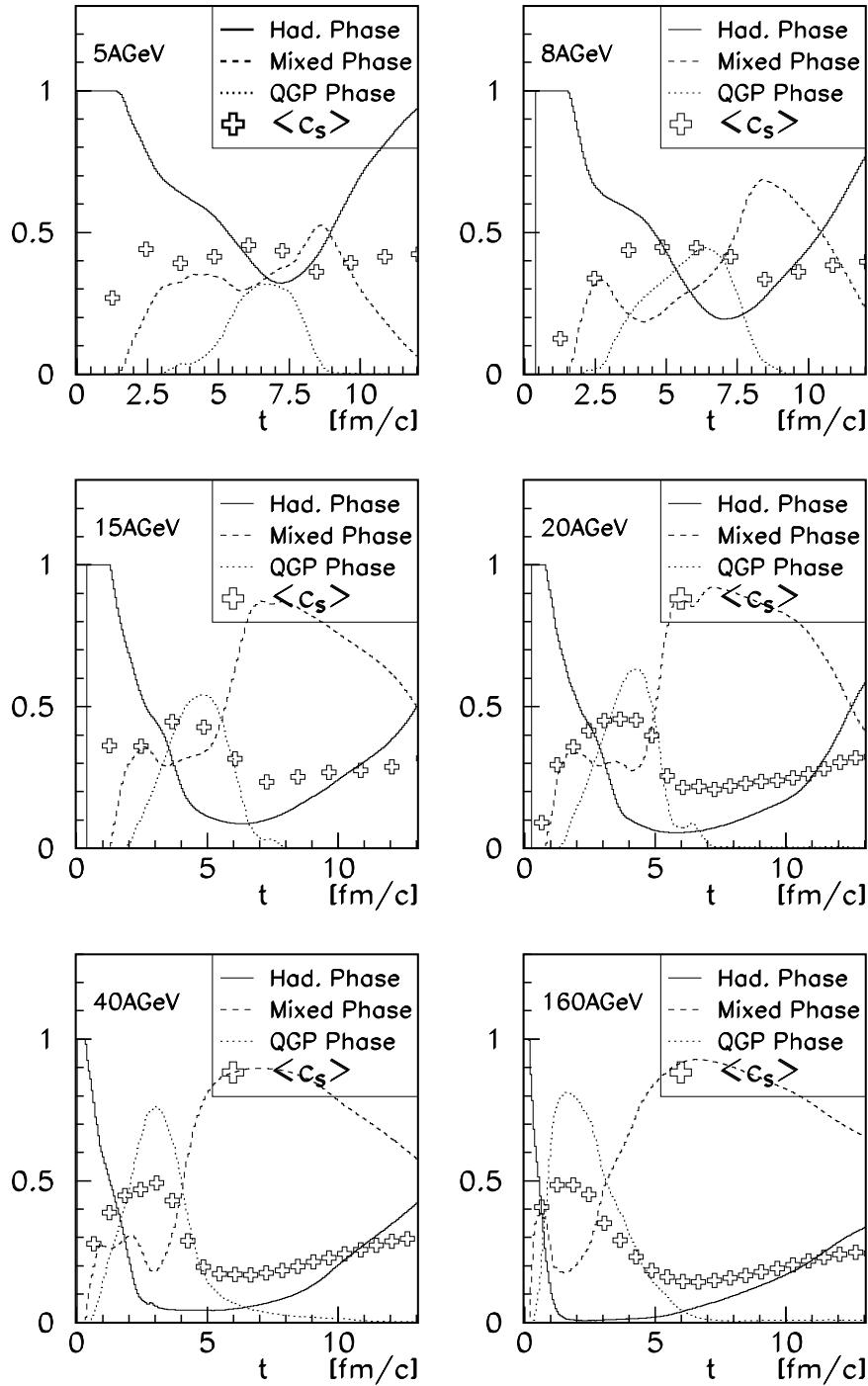


FIG. 1. Evolution of the isentropic speed of sound and the fraction of baryon charge in the various phases; Pb+Pb collisions at $b = 3$ fm.

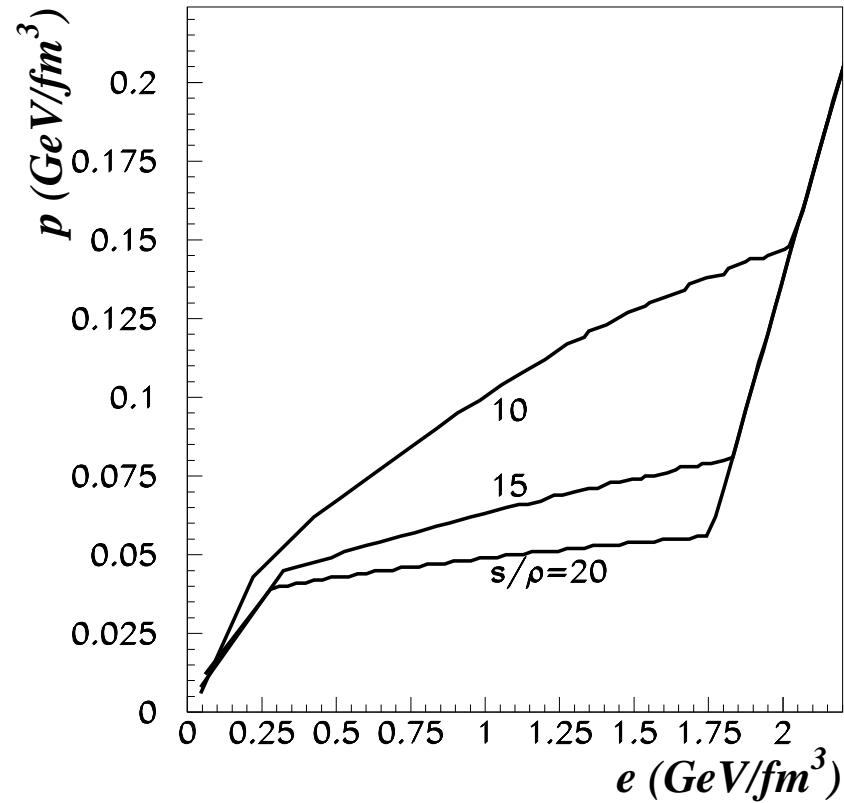


FIG. 2. Pressure as a function of energy density for various values of the entropy per net baryon. $s/\rho = 10, 20$ corresponds to $E_{Lab}^{kin} = 10A$ GeV and $40A$ GeV, respectively.

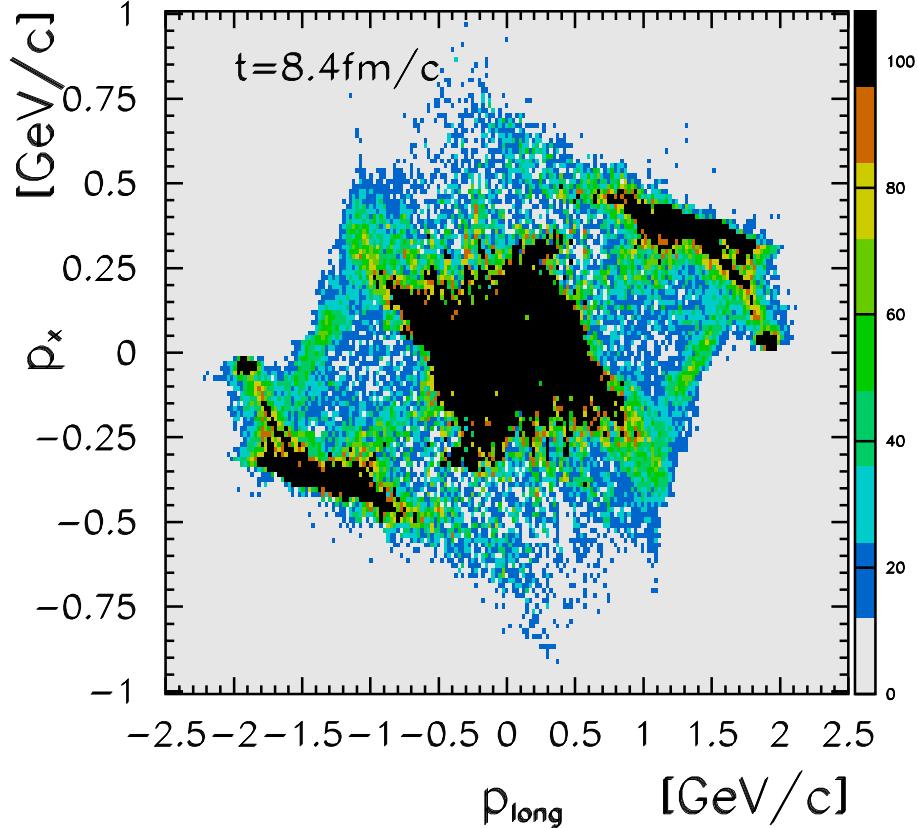


FIG. 3. Distribution of the time-like component of the net-baryon four-current in momentum space ($p_x - p_{long}$ plane). The highest contour-level is at 1/30th of the maximum, to offer a clearer view of the midrapidity region. Pb+Pb collisions at $b = 3$ fm, $E_{Lab}^{kin} = 8A$ GeV.

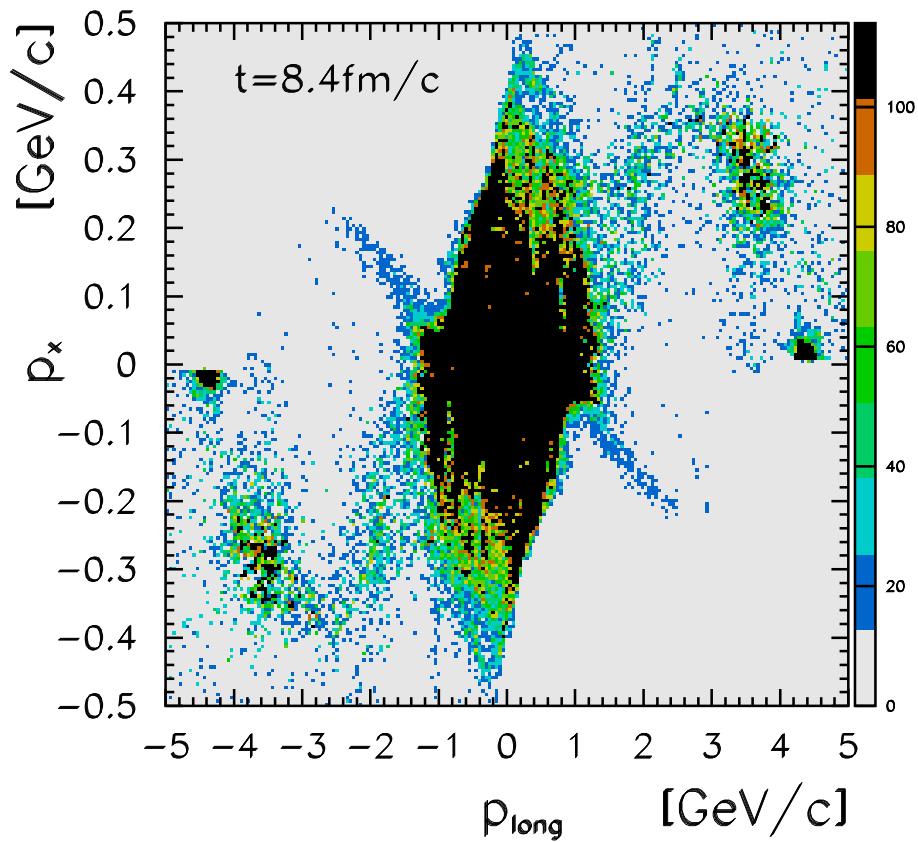


FIG. 4. As in Fig. 3 but for $E_{\text{Lab}}^{\text{kin}} = 40A \text{ GeV}$.